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Entanglement and nonlocality in a coupled-cavity system

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No instrument is able to measure directly the quantum entanglement of a system. However, both theory and experiment, following the well-known Bell inequality, reveal the existence of the entanglement phenomenon in quantum mechanics. To examine the characterization of quantum entanglement, here we present a two-site cavity system, in which each cavity contains a Λ-type three-level atom and the two sites are identical and coupled with each other. We investigate and calculate the bipartite entanglement entropy of the system for the ground states. For photons of different types, corresponding to the two ground states of the atom, we investigate the correlations and violation of the Bell inequality.

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1. INTRODUCTION

The Bell inequality had been raised in the field of quantum entanglement in 1964 [1]. Measuring some different but coupled parts of an isolated system, we definitely obtain some statistical correlations. Unlike classical correlations, which satisfy the Bell inequality and are in consistence with our perceptions, the quantum correlations calculated following Bell’s method that, taking into consideration the entangled states, just revealed some phenomena that are incompatible with classical correlations. From then until now, violations of the Bell inequality have been observed in a large number of experiments, which prove the existence of quantum entanglement [2–7].

Investigating the entanglement of a system helps us to understand the complex phenomenon of nonlocality, which can be exhibited via violation of the Bell inequality. Quantum entanglement brings some new features to our common knowledge about the world that are necessary for nonlocality. These features make quantum entanglement essential to quantum information science.

Entanglement is so important that we need a careful study about its features. Coupled-cavity systems, allowing the manipulation of states of quantum systems, are a kind of perfect candidate for our study of quantum entanglement. Nowadays, various methods for preparing and controlling the states have been demonstrated, among which are two-level atoms or artificial atoms with a two-level energy structure [8–15]. Photons, interacting with atoms in cavities and hopping between adjacent cavities, act as information carriers. To enhance the ability of transporting and controlling information, more complex systems have been studied, such as employing more energy levels to take part in the interaction with cavity modes or adding more atoms in a cavity [16,17]. In addition, Ji et al. investigated the Josephson effect for photons in two weakly linked microcavities [18], the non-Abelian Josephson effect between two F = 2 spinor Bose–Einstein condensates in double optical traps [19], and the quantum magnetic dynamics of polarized light in arrays of microcavities [20]. In detail, more degrees of freedom for photons are needed to get involved in the interaction with atoms. Here we consider the case where a Λ-type three-level atom is embedded in each of the two cavities.

The probability distribution for the bare states of the ground state depends on different parameters. The intersect states, the pure atomic state, and all photon states play different roles according to different parameters. Thus, they will be connected with the entropy of the system. We explore in detail their correlations. These correlations are used in the following discussion on bipartite entanglement entropies of the system, based on the von Neumann entropy method. To reveal the entanglement between subparts of subsystems in the whole system, we investigate the nonlocality, in which the Clauser–Horne–Shimony–Holt (CHSH) inequality [21] is checked, implying that the incompatibility definitely indicates a high entanglement.

The paper is organized as follows. In Section 2, we give our model and the matrix expression of the Hamiltonian of the
system. In Section 3, we study the entropies of the ground states over a wide range of hopping and detuning. In Section 4, we study the bipartite entanglement entropies of the system for the ground state. The nonlocality of the system is discussed in Section 5. Finally, we give a summary in Section 6.

2. CALCULATING FOR THE HAMILTONIAN

First, we introduce the two-site coupled cavity array system with Λ-type three-level atoms. As is shown in Fig. 1, the system consists mainly of two cavities; embedded in each cavity is a Λ-type three-level atom. The ground states of the atom, |H⟩ and |V⟩, are degenerate, where H and V indicate the horizontally and vertically polarized states, respectively.

When the atom in the excited state jumps to one of two degenerate ground states, an H- or V-polarized photon is emitted. For the case of emission of an H-polarized photon, it would interact with the atom located in the |H⟩ state, stimulate the atom up to the excited state, stay in the cavity, or hop to the adjacent cavity. The Hamiltonian for the two-cavity system, \( H = H_c + H_s + H_j \), consists of three parts, where \( H_c \) is a cavity Hamiltonian representing the harmonic oscillation parts for both cavities, \( H_s \) is the atomic Hamiltonian, and \( H_j \) is the atom–cavity interaction part. The three parts of the Hamiltonian can be rewritten as follows (setting \( \hbar = 1 \)):

\[
\hat{H}_c = \omega_c \sum_{j=1,2} \sum_{i=H,V} \hat{a}_{j,i}^\dagger \hat{a}_{j,i} + A \sum_{i=H,V} (\hat{a}_{1,i}^\dagger \hat{a}_{2,i}^\dagger + \hat{a}_{1,i} \hat{a}_{2,i}),
\]

\[
\hat{H}_s = \omega_s \sum_{j=1,2} |\phi_j\rangle \langle \phi_j|,
\]

\[
\hat{H}_j = \sum_{j=1,2} \sum_{i=H,V} (g \hat{a}_{j,i} |\phi_j\rangle \langle \phi_j| + g \hat{a}_{j,i}^\dagger |\phi_j\rangle \langle \phi_j|).
\]

Here, \( \omega_c \) and \( \omega_s \) are the cavity and atom frequencies, \( g \) (\( s = H \) or V for different polarized photons) is the atom–cavity coupling strength for the ground states of the atoms, and \( A \) is the hopping strength between the cavities for the photons. The operator \( \hat{a}_{j,i} \) (\( \hat{a}_{j,i}^\dagger \)) is the annihilation (creation) operator associated with the field in cavity \( j (j = 1, 2) \) for the polarization \( s = H, V \). The operator \( |\phi_j\rangle \langle \phi_j| \) (|\phi_j\rangle \langle \phi_j|) is the atomic lowering (raising) operator in cavity \( j \) for the polarization \( s = H, V \). The Jaynes–Cummings model under the rotating-wave approximation is involved to describe the system below.

To obtain the matrix expression of the Hamiltonian, we consider a subspace of the two-excitation space, where all the states can freely evolve into another one through one or more steps. We represent a general form of the state in the following way: \( |\phi⟩ = |m_j⟩_H |n_j⟩_V |p_j⟩_H \otimes |m_j⟩_V |n_j⟩_V |p_j⟩_V \), where \( m_j \) denotes the number of photonic excitations with polarization \( H \) in cavity \( j \), and thus \( |m_j⟩_H \) denotes the photonic state for \( H \)-polarized photons in cavity \( j \). Similarly, \( |n_j⟩_V \) denotes the photonic state for \( V \)-polarized photons in cavity \( j \). The atomic excitation is denoted as \( |p_j⟩_p \), where \( p = H, V, e \) represent the two ground states and the excited state for the atom in cavity \( j \). Thus, the bare states of the system can be written as follows:

(i) Photonic states when no atom is excited:

\[
|\phi_{11}⟩ = |2⟩_H |0⟩_V |0⟩_V \otimes |0⟩_H |0⟩_V |0⟩_H, \]

\[
|\phi_{21}⟩ = |1⟩_H |1⟩_V |0⟩_V \otimes |0⟩_H |0⟩_V |1⟩_H, \]

\[
|\phi_{22}⟩ = |1⟩_H |1⟩_V |1⟩_V \otimes |0⟩_H |0⟩_V |V⟩_H, \]

\[
|\phi_{31}⟩ = |1⟩_H |0⟩_V |0⟩_V \otimes |1⟩_H |0⟩_V |0⟩_H, \]

\[
|\phi_{32}⟩ = |1⟩_H |0⟩_V |1⟩_V \otimes |1⟩_H |0⟩_V |1⟩_H, \]

\[
|\phi_{33}⟩ = |0⟩_H |2⟩_V |0⟩_V \otimes |0⟩_H |0⟩_V |0⟩_H, \]

\[
|\phi_{41}⟩ = |0⟩_H |1⟩_V |0⟩_V \otimes |1⟩_H |0⟩_V |V⟩_H, \]

\[
|\phi_{42}⟩ = |0⟩_H |1⟩_V |1⟩_V \otimes |1⟩_H |0⟩_V |1⟩_H, \]

\[
|\phi_{43}⟩ = |0⟩_H |1⟩_V |0⟩_V \otimes |1⟩_H |1⟩_V |0⟩_H, \]

\[
|\phi_{51}⟩ = |0⟩_H |0⟩_V |0⟩_V \otimes |0⟩_H |0⟩_V |V⟩_H, \]

\[
|\phi_{52}⟩ = |0⟩_H |0⟩_V |1⟩_V \otimes |0⟩_H |0⟩_V |1⟩_H, \]

\[
|\phi_{53}⟩ = |0⟩_H |0⟩_V |0⟩_V \otimes |0⟩_H |0⟩_V |1⟩_H, \]

(ii) Pure atomic states when both of the two atoms are excited:

\[
|\phi_{61}⟩ = |0⟩_H |0⟩_V |0⟩_V \otimes |0⟩_H |0⟩_V |0⟩_V, \]

\[
|\phi_{62}⟩ = |0⟩_H |0⟩_V |0⟩_V \otimes |0⟩_H |0⟩_V |0⟩_V, \]

\[
|\phi_{63}⟩ = |0⟩_H |0⟩_V |0⟩_V \otimes |0⟩_H |0⟩_V |0⟩_V, \]

\[
|\phi_{64}⟩ = |0⟩_H |0⟩_V |0⟩_V \otimes |0⟩_H |0⟩_V |0⟩_V, \]

\[
|\phi_{65}⟩ = |0⟩_H |0⟩_V |0⟩_V \otimes |0⟩_H |0⟩_V |0⟩_V, \]

(iii) Intersect states, where there is one atomic excitation and one photonic excitation:

\[
|\phi_{71}⟩ = |1⟩_H |0⟩_V |0⟩_V \otimes |0⟩_H |0⟩_V |0⟩_H, \]

\[
|\phi_{72}⟩ = |0⟩_H |1⟩_V |0⟩_V \otimes |0⟩_H |0⟩_V |0⟩_V, \]

\[
|\phi_{73}⟩ = |0⟩_H |0⟩_V |0⟩_V \otimes |1⟩_H |0⟩_V |0⟩_H, \]

\[
|\phi_{74}⟩ = |0⟩_H |0⟩_V |0⟩_V \otimes |0⟩_H |1⟩_V |0⟩_V, \]

\[
|\phi_{75}⟩ = |1⟩_H |0⟩_V |0⟩_V \otimes |0⟩_H |0⟩_V |0⟩_H, \]
\[ |\phi_6\rangle = |0\rangle_H |1\rangle_V |V\rangle_c \otimes |0\rangle_H |0\rangle_V |e\rangle, \tag{24} \]
\[ |\phi_7\rangle = |0\rangle_H |0\rangle_V |H\rangle_c \otimes |1\rangle_H |0\rangle_V |e\rangle, \tag{25} \]
\[ |\phi_8\rangle = |0\rangle_H |0\rangle_V |V\rangle_c \otimes |0\rangle_H |1\rangle_V |e\rangle. \tag{26} \]

In practice, we are not so interested in the actual eigen energies of the subspace, we are definitely going to get its matrix form. By setting \(A\) to make it look more compact for the layout. By setting \(\omega = .0136\), we can yield the results represented in Refs. [10,11]. For now, the system, referring to an idea that is familiar in information theory, we introduce the additional photon with a different polarization, rather than the photons to be in the excited states.

Meanwhile, for small hopping, there is a preference for the photons to stay in their own cavities, interacting with atoms or being in their eigenmodes, rather than hopping to the adjacent cavities. In these situations, the probabilities for states described by Eqs. (4)–(17) and states represented by Eqs. (19)–(26) must be very small, while the probability for the atomic state defined by Eq. (18) must be very large. The disorder of this probability distribution is so trivial that we are not confused about determining which state would be detected.

Next, we show the probability plot for the pure atomic states in the ground states, as shown in Fig. 2. For the large negative detuning, where \(\Delta = \omega_e - \omega_b < 0\), the atomic energy level is much smaller than the energy of the photon. As a result, in the ground states, we are more likely to detect the atoms rather than the photons to be in the excited states.

Where \(\Delta = \omega_e - \omega_b\) denotes the detuning, and we have replaced \(\sqrt{2g_{H}}, \sqrt{2g_{V}}, \text{and } \sqrt{2A}\), by \(g'_{H}, g'_{V}, \text{and } A'\), respectively, to make it look more compact for the layout. By setting one of the atom–cavity coupling strengths, \(g_{H}\) or \(g_{V}\), to be zero, we can yield the results represented in Refs. [10,11]. For now, we introduce the additional photon with a different polarization; our system exhibits some new features and has more options for the manipulation of the system.

### 3. Entropy of the Ground State

In this section, we discuss the entropy for the ground state of the system, referring to an idea that is familiar in information theory. As is well known, entropy is often referred to as the number of possible arrangements or the degree of disorder. Here, we use this property to discuss the evolution of bare states for the ground states.

At first, let us examine the distributions of the probabilities of the bare states in ground states for different parameters, \(\Delta\) and \(A\). We can calculate the entropy \(S = -\sum_{i=1}^{n} p_i \log_2 p_i (\sum_{i=1}^{n} p_i = 1)\), which is well known in information theory. Assuming the ground state of the system to have the form \(|\psi_i\rangle = \sum_{j=1}^{23} C_i |\phi_j\rangle\), the entropy of this state is thus \(S = -\sum_{j=1}^{23} |C_i|^2 \log_2 |C_i|^2\).

A contour plot of \(S(\Delta, A)\) is shown in Fig. 2. For the large negative detuning, where \(\Delta = \omega_e - \omega_b < 0\), the atomic energy level is much smaller than the energy of the photon. As a result, in the ground states, we are more likely to detect the atoms rather than the photons to be in the excited states.
cavities. Before hopping to the adjacent cavity, they become more likely to exist in their eigenstates rather than annihilating and raising the atoms, sharing with the photons the same cavities, up to the excited states. As a result, the probabilities for pure atomic states decrease, while the probabilities for intersect states increase. The intersect states indicating that the atomic excitation and photonic excitation appears in different cavities, are more likely to be detected than the states in which both the excitations are in the same cavities.

We show in Figs. 4 and 5 that the probabilities for Eqs. (21)–(24) are much more than those of Eqs. (19), (20), (25), and (26) in the ground states. This is in agreement with our analyses.

When the hopping strength goes up to a certain level, around the line represented by $A = -\Delta$, things begin to change. Even though the atomic energy level is still much smaller than the energy of the photon for the same $\Delta$, the interacting strength between atom and photon is much smaller than the hopping strength. As a result, the chances for us to detect the photonic states and the atomic states draw near. This creates chaos around the $A = -\Delta$, which obtains the high entropy region. With continued increase of the hopping strength, the chaos gradually declines, implying that the chances for detecting the system located in the atomic state or in the intersect states becomes dimmer, while the all-photon
states become the main part of the ground state, as shown in Fig. 6. Because of a larger number of possible bare states of the all-photon states than that of the atomic states, the entropy remains at a pretty high level for the large positive hopping strength, which has been checked in Fig. 2.

As the detuning increases, the atomic energy levels become smaller, while the photon energy gets higher. For the ground states, the probability for us to find the photons keeps on going up while the probability for the atoms keeps on going down. Because of this, the probability of the intersect states gets bigger, which raises the entropy of the system. The peak of entropy appears around the $\Delta = \Delta_g$ region. When we keep increasing the detuning, the opportunity for detecting the atoms becomes even smaller, thus the probabilities for the intersect states grow smaller. The all-photon states begin to take up the main parts of the ground states. As a result, the entropy goes down.

During the evolution process discussed above, the different polarized photons play different roles. We take the intersect states as an example, as shown in Fig. 7. Having set $g_H = 2g_V$, the coupling strength between an $H$-polarized photon and the atom is stronger than that for a $V$-polarized photon. As a result, in the ground state, we are more likely to detect an $H$-polarized photon rather than a $V$-polarized photon.

In this section, we discussed mainly the atom–atom, atom–photon, and photon–photon evolution for different parameters of the system. In analyzing the physical processes, we calculated the probability distributions for different part of the bare states. Results show that entropy for the bare state reflects the evolution or phase transformation quite well.

**4. BIPARTITE ENTANGLEMENT**

We discuss the bipartite entanglement about the ground state of the system, in which the von Neumann entropy method is employed. We assume that a system is composed of two parts, $A$ and $B$, and is represented as $\rho_{AB}$ in density matrix form. Given the reduced density matrices, $\rho_A = \text{Tr}_B(\rho_{AB})$ and $\rho_B = \text{Tr}_A(\rho_{AB})$, the entanglement between $A$ and $B$ can be expressed as $S(\rho_A) = -\text{Tr}(\rho_A \log_2 \rho_A)$ or $S(\rho_B) = -\text{Tr}(\rho_B \log_2 \rho_B)$.

Let us first look at an example. For the subspace composed of the $H$-polarized photons in one cavity, we may obtain three different results (0 photon, 1 photon, and 2 photons) when a measurement is performed. We write these states as $|0\rangle = (1, 0, 0)^T$, $|1\rangle = (0, 1, 0)^T$ and $|2\rangle = (0, 0, 1)^T$, respectively. These bare states are also the eigenstates of the density matrix for this part that is reduced from the whole. We can explicitly measure the probabilities for this subsystem to be in these three states. We will use these results in the following for discussing their entropies. The measurement operators for this subspace can be defined as

$$
\hat{M}_0 = \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix},
$$

$$
\hat{M}_1 = \begin{pmatrix}
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{pmatrix},
$$

$$
\hat{M}_2 = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix}.
$$

If the ground state of the system is $|\psi\rangle$, then the probability that outcome $m (m = 0, 1, 2)$ occurs is given by

$$
P_m = \langle \psi | \hat{E}_m \hat{E}^\dagger_m | \psi \rangle,
$$

where $\hat{E}_m = \hat{M}_m \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1$, and $\hat{I}_3$ is a $3 \times 3$ identity matrix that represents the operator on other parts of the whole system. For the smaller hopping strength, we obtain the probability distribution for this subsystem, as shown in Fig. 8.

For large negative detuning, the photon excitation, which has higher energy than the atomic excitation, is less likely to be detected in the ground states. The probability for detecting any $H$-polarized photon is almost zero. As the detuning increases, the intersect states and the later all-photon states gradually take up the main parts in the ground states. We can find the growth trend in the line for the probability of one $H$-polarized photon. This line rises ahead of the line for two $H$-polarized photons. This is because, for the small detuning, the probability for the photon is low and the small hopping strength more or less prevents the photons from hopping into the adjacent cavity, which may be described as a two-photon state in this case. Finally, for the large positive detuning, the photon excitations carry smaller energy and are more likely to be detected in the ground states. The probability for detecting the $H$-polarized two-photon states increases, while the probability for the $H$-polarized single-photon states decreases. Because of the existence of the $V$-polarized photons, it is still possible to detect the non-photon state even for the very large detuning. This is what we have already learned from Section 3. We now learn from Fig. 8 that, for the small hopping, there is no difficulty in determining which state the $H$-polarized photon subsystem in one cavity is in, and the entropy is almost zero. As the
detuning increases, it is more difficult to distinguish between them, which leads to large entropy, as shown in Fig. 9.

For the subsystems, such as $H$-polarized photons in one cavity, the $V$-polarized photons in one cavity, and the one-atom system, their eigenstates are the same as the bare states. As discussed in Section 3, around the $A = -\Delta$ region, the entropies of the bare states for the whole system reach their peaks. The probability distributions for the states of these subspaces turn into the most chaotic situations. We can see from Fig. 9 that their entropies reach the maxima, which means strong entanglement. We also confirm that, for the subsystems, such as the $H$-polarized photons in two cavities, the $V$-polarized photons in two cavities, and the two-atom system, their eigen states are quite similar to the bare states. Their behaviors of entanglement are just the same as we expected—they first go up and reach their peaks around the point of $A = -\Delta$, and after that go down to some steady levels.

However, the one-site entropy, which measures the entanglement between site I (including the photons and the atom) and site II, remains at a quite low level (be zero in this figure at the point of $A = -\Delta$) and after that rise to a high level. This is quite strange, so we need a more detailed understanding. Calculations show that this subsystem is mainly composed of three eigenstates, $|\psi_1\rangle = C_1|00e\rangle + C_2|10H\rangle + C_3|01V\rangle$, $|\psi_2\rangle = C_1|10e\rangle + C_2|11V\rangle + C_3|20H\rangle$, and $|\psi_3\rangle = C_1|00H\rangle$, where $C_i$ are coefficients and satisfy $\sum_i|C_i|^2 = 1$ for each state. We can find from Eqs. (4)–(26) that, if one site is in the state $|\psi_1\rangle$, the other site must be in the state

![Fig. 7. Probabilities of the intersect states: (a) one atom excited in one cavity and one $H$-polarized photon excited in the other cavity, (b) one atom excited in one cavity and one $V$-polarized photon excited in the other cavity, (c) one atom and one $H$-polarized photon excited in the same cavity, and (d) one atom and one $V$-polarized photon excited in the same cavity.](image-url)
Probability distributions are shown in Fig. 10. The intersect being empty. When the detuning strength is large enough, states indicating two excitations in one cavity and the other indicating one excitation per cavity is higher than that of the other. Thus the probability of the state indicating the two excitations per cavity is higher than that of the other. The inset shows the total probability for the three eigenstates. Parameter values are $g_H = 2g_V$ and $A = 0.01g_H$.

Fig. 8. Probability distribution of the $H$-polarized photons under the small hopping limit. Parameter values are $g_H = 2g_V$ and $A = 0.01g_H$.

Fig. 9. Bipartite entanglement under the small hopping limit. Parameter values are $g_H = 2g_V$ and $A = 0.01g_H$.

Fig. 10. Probability distribution of the eigenstates in the one-site subsystem. The inset shows the total probability for the three eigenstates. Parameter values are $g_H = 2g_V$ and $A = 0.01g_H$.

the probabilities of the eigenstates are comparable. Now, it is clear why the one-site entropy looks as shown in Fig. 9.

Figure 11 shows the entropies under the large hopping limit. As discussed above, the ground states of the system for the large negative detuning are most likely to be in the pure atomic states. The probability distributions in this manner are of quite low disorder, and it is destined to be zero. Around the point of $A = -\Delta$, all the entropies reach their maxima. After that, the probabilities of the pure atomic state and the intersect states get to some quite low level and the all-photon states play the main role, which maintains the entropies in quite high and steady levels.

5. NONLOCALITY: CHSH INEQUALITY

In Section 4, we have discussed some bipartite entanglements of the system. However, the entanglement between two subsystems in one part of the system (for example, photons in cavity I and photons in cavity II) is still unknown. It is very hard for us to tell if there is entanglement between them. In this section, we will employ the CHSH inequality [21] to study the properties of entanglement between these subsystems.

First, following the method reported in Refs. [21,22], let us measure the number of $H$-polarized photons in cavity I; the results would be 0, 1, and 2. If no photon is detected, our measuring apparatus would respond as $-1$, while if any photon is detected, the response would be $+1$. The $V$-polarized photons can be measured in the same way. We assume that the polarizations of two different photons are orthogonal, as $x$ and $y$ directions. For cavity II, we measure the number of photons in some orthogonal directions, excluding $x$ and $y$, labeled as $\alpha$ and $\beta$, corresponding to the $x$ direction, in which $\beta = \alpha + \pi/2$. In each experiment, we choose two directions $(a, b)$, where $a \in [x, y]$ and $b \in [\alpha, \beta]$, and would obtain two results $M$ and $N$, where $M, N \in [-1, 1]$, respectively. Carrying on the experiment a large number of times, for a specific pair of directions $(a, b)$, we would obtain the expectation value of the product $MN$ for given measurement choices $(M_aN_b) = \sum_{M,N} MNp(MN|ab)$, where $p(MN|ab)$ indicates the probability measuring the system in directions $a$ and $b$, respectively, and getting the responses of $M$ and $N$. Thus we would obtain the famous CHSH inequality

\[ S = \langle M_xN_a \rangle + \langle M_xN_b \rangle + \langle M_yN_a \rangle - \langle M_yN_b \rangle \leq 2. \] (30)
We check this inequality in the ground states for our system in the classical and quantum manners. The quantum measurement is performed as follows.

For the $H$- and $V$-polarized photons in cavity I, the eigenstates are $|0\rangle = (1,0,0)^T$, $|1\rangle = (0,1,0)^T$, and $|2\rangle = (0,0,1)^T$, and the responses are $-1$, $+1$, and $+1$, respectively, and we define the measurement operators:

$$M_x = M_y = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (31)$$

The measurement operators in cavity II are thus

$$N_\alpha = (\cos \alpha, \sin \alpha) \cdot (M_x, M_y), \quad (32)$$

$$N_\beta = (\cos \beta, \sin \beta) \cdot (M_x, M_y). \quad (33)$$

Up to now, it is not difficult to check the CHSH inequality in Eq. (30). Figure 12 shows our results under the small hopping limit. The results calculated in the classical manner, shown by the solid curves, are in agreement with the CHSH inequality. However, the results calculated in the quantum manner with $(\alpha, \beta) = (\pi/4, 3\pi/4)$, shown by the dashed curves, are incompatible with this inequality, in particular for the large negative detuning. It indicates that a strong nonlocality lies between photons in the two cavities. We have shown in Fig. 9 that there seems to be no bipartite entanglement in the large negative detuning region. However, our results show that there exist entanglements between subparts that are part of the system. The calculated results under the large hopping limit show also the existence of nonlocality, which indicates the entanglement between photons in the two cavities.

### 6. CONCLUSIONS

We have examined mainly bipartite entanglement entropy and nonlocality, which definitely indicate the existence of entanglement in the ground state. For the entropy of the ground states, we analyzed the probability distributions of the bare states in detail over a wide range of the hopping and detuning parameters, in particular for different polarized photons.

For the bipartite entanglement entropy, the eigenstates of a small subsystem, e.g., a single $H$-polarized photon, are in agreement with their bare states. It is not difficult to show their behaviors for bipartite entanglement entropy just by referring to the entropy for ground states. However, for a larger subsystem, e.g., a single site, their behaviors become rather complicated.

For nonlocality, we also consider just the ground states of the system for different parameters. Because of entanglement,
there exists nonlocality. As the detuning changes, the entanglement between the photons in different cavities is also changed. It may become so small that the classical effect plays an important or even a main role in the experiment, which results in agreement with the CHSH inequality. Therefore, we can say that experiments agreeing with the CHSH inequality do not exclude the existence of entanglement, but results incompatible with the CHSH inequality indicate the existence of entanglement.

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**REFERENCES**